of phonon monochromator. The geometric filtering of phonon wavelengths distinguishes the present result from earlier studies of phonon interactions in double quantum dots where probe currents were passed directly through the dots[4,5].

Although QPCs have been used to study quantum dots for some time, this observation of phonon-mediated back-action has appeared now because it depends on a strong asymmetry between the charge tunnel barriers in a double-dot system, which can occur when one barrier is blocked by a third dot. Hence it is no coincidence that this result closely follows the recent report of coherent manipulation of a triple quantum dot by some of the same authors, also reported in *Nature Physics*[6]. Granger and co-workers, as well as some of the same authors in previous work[7], also report the effect in double dots biased to assure the needed asymmetry.

A key question now is whether understanding this phonon-induced back-action may allow back-action reduction to improve quantum measurements on dots. The interference effects observed give a strong indication that this may be possible: appropriate geometric and biasing conditions may enable a quantum-dot molecule to ignore the phonons originating from strongly driven QPCs by exploiting positions of destructive interference.

There is still a long way to go before quantum measurement in quantum dots reaches the extreme levels of sensitivity available to measurements of ensembles of atoms in vacuum, or more recently to nanomechanical oscillators. The understanding and reduction of phonon-mediated back-action from QPCs, however, is only one direction in which charge sensing of quantum dots is improving. As another recent example, charge transitions in double quantum dots have recently been observed by means of shifts in either the frequency or the phase of high-finesse superconducting microwave resonators[8,9]. The hybridization of quantum dots with superconducting microwave elements leads to new methods for interferometry and amplification. When such measurements reach quantum limits of sensitivity and are used on multiple quantum dots, the inherent entanglement with probe currents enables new routes for coupling distant dots together. If we do not mitigate back-action mechanisms, though, such schemes could easily fail, indicating the importance of understanding all the noise — acoustic and otherwise — from the measurement circuitry. ❐

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COMPLEX NETWORKS

The missing link

A study shows that controlling link dynamics on a network is distinctly different from controlling the dynamics of its nodes. This development illustrates how ideas from control-systems engineering can help us better understand the organization of complex systems.

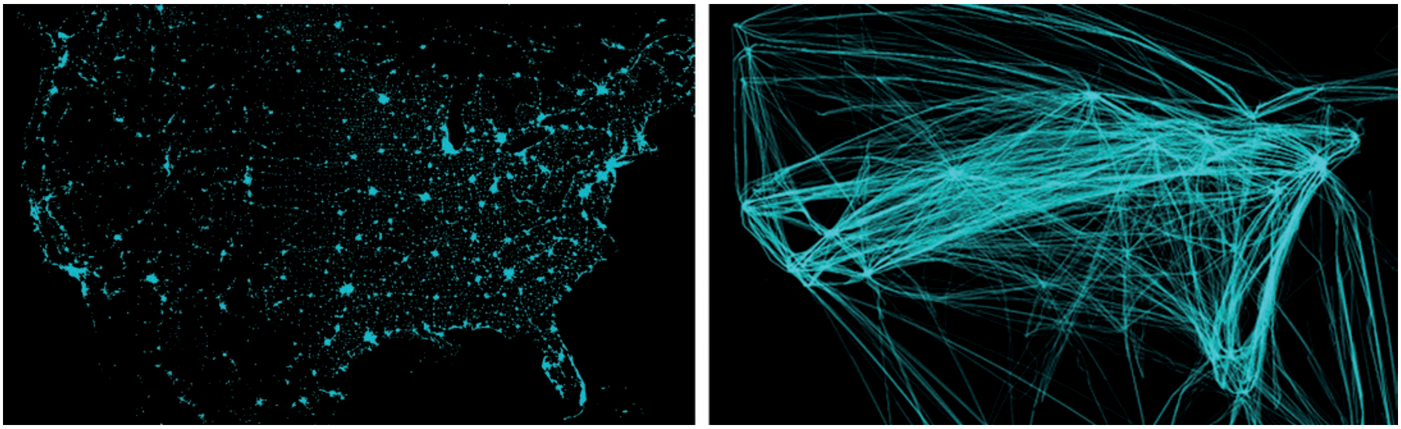
Jean-Jacques Slotine and Yang-Yu Liu

The adult human brain comprises more than 10^{11} neurons, each connected to around 10^4 synapses — meaning that the percentage of potential connections that actually form synapses is only about 0.00001%. In such sparsely connected networks, connection topology becomes paramount, and understanding how it mediates control has implications far beyond the usual applications of control theory, such as aircraft design or robotics[1,2]. One way of achieving this involves studying controllability, which concerns our ability to drive a dynamic system from any initial state to any final state in finite time[3]. Writing in *Nature Physics*, Tamás Nepusz and Tamás Vicsek have taken an important step towards understanding the controllability of complex networks based on their topology[4].

To model networks as dynamic systems, one can associate each network node with a state variable, whose time evolution depends on the state variables of the node's neighbours. Such a mapping is natural and immediate in many real systems for which the state variable of a node has a clear physical meaning, such as the concentration of a metabolite in metabolic networks or the expression level of a gene in transcriptional regulatory networks. These nodal dynamics have proven effective in revealing key insights into network controllability — including the result that sparse inhomogeneous networks are the most difficult to control — through a combination of control theory, graph theory and statistical physics[5]. This has in turn triggered further research into optimization through structural perturbation[6], the cost of control energy[7] and higher-order nodal dynamics[8].

Instead of exploring network controllability using nodal dynamics, Nepusz and Vicsek tackled the problem from a different angle, examining edge dynamics in systems for which a state variable is associated with each edge (or link) rather than each node[4]. This new point of view leads to results that are very different from those of nodal dynamics, and fundamentally enriches our understanding of network controllability in the process. Indeed, although in principle edge controllability can be understood mathematically as a particular case of nodal controllability by using a simple change of representation, it turns out to have several important properties of its own, as well as being amenable to a simple direct algorithm.

At first glance, edge dynamics can seem rather exotic — after all, the edges of complex networks may not even be physical entities. Yet, as Nepusz and Vicsek argue[4], for some real-world networks edge dynamics is the natural representation. For example, in social communication networks, a node (or say, an individual) constantly processes information received from its upstream neighbours and makes decisions that are communicated to its downstream neighbours. The information received and passed by a node can be



Network control. Controlling links is very different from controlling nodes.

represented by the state variables on its incoming and outgoing edges. The node itself acts like a switchboard, mapping the signals of the incoming edges onto those of the outgoing edges. This is also the case in a network of computers and routers on the Internet, where the edges represent physical connections and the state variables on the edges represent the amount of packet flow along a particular connection in a given direction. The switchboard-like mechanism of the nodes then corresponds to a load-balancing or routing mechanism that allows packets to reach their destination while avoiding congestion.

One way of probing a network's controllability involves finding the minimum set of driver nodes, whose time-dependent control can guide the system from an arbitrary point to anywhere else in its state space. For nodal dynamics, this optimization problem has already been solved by the minimum-inputs theorem[5], which maps it to a graph theoretical problem.

Using edge dynamics, we are still interested in identifying the minimum set of driver nodes, because to control an edge in a network one has to control the switchboard node from which the edge originates. Here we can exploit the mathematical duality between edge dynamics on a network and nodal dynamics on its line graph (in which each node corresponds to an edge, and each edge to a length-two directed path, in the original network). By applying the minimum-inputs theorem directly to the line graph, we obtain the minimum number of driver nodes for the line graph, which corresponds to the minimum number of driven edges in the original network. However, this procedure does not imply that the number of driver nodes in the original network is also minimized. Nepusz and Vicsek mapped this control problem to a graphical problem and developed an efficient algorithm to identify the minimum number of driver nodes for arbitrary complex networks with edge dynamics.

Nodal and edge dynamics are most easily compared by looking at the different roles played by highly connected nodes, or hubs, which reflect the duality between the two problems. In the case of node control, hubs do not typically act as drivers, because by sending their neighbours a common signal they create symmetries that restrict the state space that the system can explore. For link control the situation is reversed, because neighbours can receive different signals and so hubs can exploit their many connections. This results in fewer key nodes being needed to control the network, although the number of corresponding edges (and thus control signals) may be large[4]. Nepusz and Vicsek showed that heterogeneous and sparse networks have more controllable edge dynamics than homogeneous and dense networks do, in striking contrast to the case for nodal dynamics[5]. Positive correlation between in- and out-degrees of nodes enhances the controllability of edge dynamics, but it does not affect the controllability of nodal dynamics at all[9]. Conversely, adding self-edges to individual nodes enhances the controllability of nodal dynamics[9], but leaves the controllability of edge dynamics unchanged.

We conclude with two remarks. First, the controllability issue in networks is akin to the question of choosing the locations of flaps, ailerons and engines on an airplane, or similarly of designing safe system architecture in a nuclear plant. Once these key choices have been made, the design of actual control algorithms is still purely within the realm of control theory. Whether network tools can also help in this process for large complex systems will be an exciting research direction for the years to come.

Finally, a particularly interesting application of this result may arise in the context of evolution or development. It has been suggested that evolution may be based on ancient, optimized components whose connections are the main target of natural selection[10,11]. This point of view is based on extensive biological evidence of conserved core processes facilitating evolutionary variation[10], and independently on the need to preserve stability and functionality when aggregating stable functional subsystems[11]. From this perspective, controllability of evolution and development is primarily a link-based concept, potentially giving our current understanding of edge[4] and nodal[5] dynamics profound new implications. ❐

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